APPENDIX J

Gear Terminology and Contact-Ratio Analysis

In this Appendix, we summarize the mathematical relationship(s) between various quantities associated with spur-gears (see Figure J.1).

J.1 Nominal Spur-Gear Quantities

Various quantities associated with a spur-gear set are summarized in Table J.1 below, where:

- Column 1 is the name of the quantity as described in Chapter 15.
- Column 2 is the associated symbol (see conventions below).
- Column 3 is the unit (if any) associated with the quantity.
- Column 4 is the relationship of the given quantity with previously defined quantities.
- Column 5 is a specific case study (see example below).

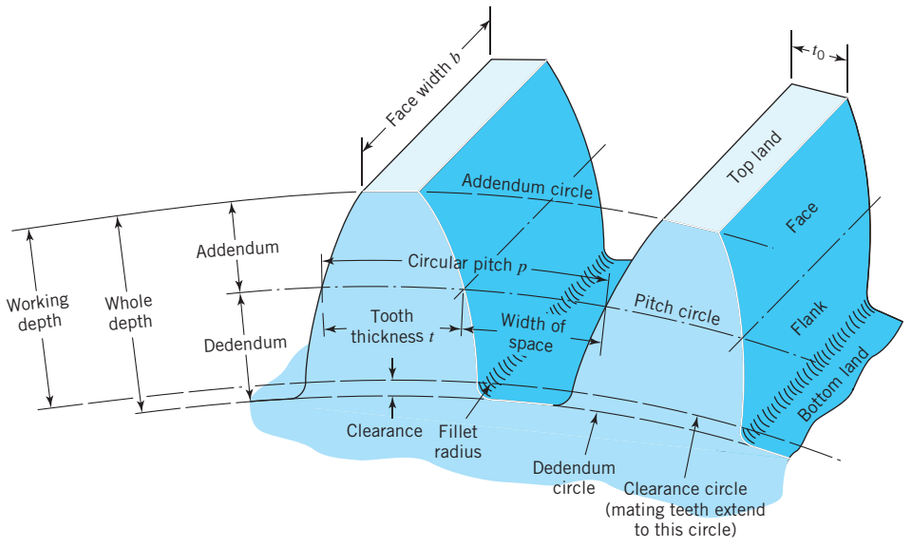

FIGURE J.1
Nomenclature of gear teeth

As in Chapter 15, we use the following conventions:

- A subscript p is used to refer to the smaller gear (pinion), while g is used to refer to the larger gear. Thus, for example, N_p is the number of teeth in the pinion, while N_g is the number of teeth in the larger gear. For quantities such as the pressure angle ϕ that are common to pinion and gear, subscripts are not used.
- In addition, the subscript b is used for base radius, a for addendum radius, and d for dedendum radius. Thus, for example, the symbol r_{bp} is the pinion base radius, while r_{ag} is the gear addendum radius.

Now consider a gear-set that consists of a 16-tooth pinion and a 40-tooth gear; diametral pitch is 2, with a nominal pressure angle of 20°. In the fifth column of Table J.1, we have entered this data in the first three rows, while in the remaining rows of the fifth column, we have computed the quantities using the relationship provided. Observe in the table that we have used the standard addendum radii for the pinion and gear, since they are less than the corresponding maximum allowable addendum radii. Given the addendum radii, one arrives at the nominal contact ratio of 1.58; these calculations are most conveniently carried out using an Excel® spreadsheet.

Table J.1 ***Nominal Quantities Associated with a Gear-Set***

Name	Symbol	Units	Relationship	Example
Number of teeth	N_p N_g			$N_p = 16$ $N_g = 32$
Nominal Diametral pitch	P	per in.		$P = 2$
Pressure angle	ϕ	degrees		$\phi = 20°$
Gear ratio	η	-	$\eta = N_g/N_p$	$\eta = 2$
Base width	b	in.	$9/P < b < 14/P$	$4.5 < b < 7$
Nominal pitch radius	r_p r_g	in.	$r_p = N_p/(2P)$ $r_g = N_g/(2P)$	$r_p = 4$ $r_g = 8$
Base radius	r_{bp} r_{bg}	in.	$r_{bp} = r_p \cos\phi$ $r_{bg} = r_g \cos\phi$	$r_{bp} = 3.76$ $r_{bg} = 7.52$
Center distance	c	in.	$c = \dfrac{(N_p + N_g)}{(2P)}$	$c = 12$
Maximum addendum radius to avoid interference	r_{ap}^{max} r_{ag}^{max}	in.	$r_{ap}^{max} = \sqrt{r_{bp}^2 + c^2 \sin^2\varphi}$ $r_{ag}^{max} = \sqrt{r_{bg}^2 + c^2 \sin^2\varphi}$	$r_{ap}^{max} = 5.57$ $r_{ag}^{max} = 8.56$
Addendum radius	r_{ap} r_{ag}	in.	$r_{ap} = (N_p + 2)/(2P)$ $r_{ag} = (N_g + 2)/(2P)$ (Standard)	$r_{ap} = 4.5$ $r_{ag} = 8.5$

Dedendum radius	r_{dp} r_{dg}	in.	$r_{dp} = (N_p - 2.5)/(2P)$ $r_{dg} = (N_g - 2.5)/(2P)$ (Standard)	$r_{dp} = 3.375$ $r_{dg} = 7.375$
Module	m	mm	$m = 25.4/P$	$m = 12.7$
Circular pitch	p	in.	$p = \pi/P$	$p = 1.57$
Tooth thickness	t	in.	$t = \pi/(2P)$	$t = 0.785$
Contact ratio	CR	-	$\Delta_p = \sqrt{r_{ap}^2 - r_{bp}^2}$ $\Delta_g = \sqrt{r_{ag}^2 - r_{bg}^2}$ $CR = \dfrac{\Delta_p + \Delta_g - c \sin\phi}{p \cos\phi}$	$\Delta_p = 2.70$ $\Delta_g = 3.96$ $CR = 1.58$

J.2 Actual Quantities

The quantities in Table J.1 are nominal quantities, i.e., they are valid when the two gears are at the nominal (ideal) center distance. In practice, the center distance will be larger than the nominal center distance, and many of the above quantities will change. This is captured in Table J.2.

In Table J.2, we identify the specific quantities that change when the center distance increases. Quantities such as number of teeth and gear ratio do not depend on the center distance are therefore not listed below.

In order to distinguish actual quantities from their nominal counterparts in Table J.1, we use a bar above the symbol in Table J.2. For example, since c is the nominal center distance, $\overline{c}$ is the actual center distance. Similarly, $\overline{\phi}$ is the actual pressure angle.

Continuing with the example of the previous section, suppose the distance between the gear-centers is 0.05 inches more than the nominal; what is the contact ratio and backlash?

In the fifth column of Table J.2, we have entered the data provided in the first row, while in the remaining rows, the quantities are computed using the relationships provided. Observe that the contact ratio has dropped from 1.58 in Table J.1 to 1.49 in Table J.2.

Table J.2 ***Contact Ratio and Other Quantities at an Actual Center Distance***

Name	Symbol	Units	Relationship	Example
Actual working distance	$\overline{c}$	in.		$\overline{c} = 12.05$
Ratio of center distances	λ		$\lambda = \overline{c}/c$	$\lambda = 1.004$
Actual pressure angle	$\overline{\phi}$	degrees	$\cos\overline{\phi} = \cos\phi/\lambda$	$\overline{\phi} = 20.64°$

(Continued)

Actual pitch	$\overline{P}$	per in.	$\overline{P} = P/\lambda$	$\overline{P} = 1.99$
Actual pitch radius	$\overline{r}_p$ $\overline{r}_g$	in.	$\overline{r}_p = r_p\lambda$ $\overline{r}_g = r_g\lambda$	$\overline{r}_p = 4.0167$ $\overline{r}_g = 8.0333$
Actual circular pitch	$\overline{p}$	in.	$\overline{p} = p\lambda$	p = 1.5773
Maximum addendum radius to avoid interference	$\overline{r}_{ap}^{max}$ $\overline{r}_{ag}^{max}$	in.	$\overline{r}_{ap}^{max} = \sqrt{r_{bp}^2 + \overline{c}^2 \sin^2 \overline{\varphi}}$ $\overline{r}_{ag}^{max} = \sqrt{r_{bg}^2 + \overline{c}^2 \sin^2 \overline{\varphi}}$	$\overline{r}_{ap}^{max} = 5.67$ $\overline{r}_{ag}^{max} = 8.63$
Actual contact ratio	$\overline{CR}$	-	$\Delta_p = \sqrt{r_{ap}^2 - r_{bp}^2}$ $\Delta_g = \sqrt{r_{ag}^2 - r_{bp}^2}$ $\overline{CR} = \dfrac{\Delta_p + \Delta_g - \overline{c} \sin \overline{\phi}}{\overline{p} \cos \overline{\phi}}$	$\overline{CR} = 1.49$
Backlash (measured on pitch circle)	$\overline{B}$	in.	$\overline{B} = 2(\overline{c} - c) \tan \overline{\phi}$	$\overline{B} = 0.0377$

J.3 Illustrative Example

We now rework Sample Problem 15.1D (reproduced below for convenience) using the two tables above.

Sample Problem 15.1D

Two parallel shafts with (nominal) 4-in. center distance are to be connected by 6-pitch, 20° spur gears providing a velocity ratio of –3.0. (a) Determine the pitch diameters and numbers of teeth in the pinion and gear. (b) Determine whether there will be interference when standard full-depth teeth are used. (c) Determine the contact ratio.

Solution: Observe that, in this example, the number of pinion-teeth and number of gear-teeth are not provided; instead the gear ratio and the nominal center distance are provided. The first objective is to determine the number of teeth from the given data.

(**a**) From the expression for gear ratio and nominal center distance in Table J.1, we have $N_g/N_p = 3$ and $(N_p + N_g)/(2P) = 4$. Further, since the pitch P is 6, we have $N_g = 36$ and $N_p = 12$. We can now enter the data in the table below, and compute quantities, such as nominal pitch radius, etc.

(**b**) The standard addendum radius for the gear is $r_{ag} = (N_g + 2)/(2P) = 3.17$. Since this is greater than the maximum allowable addendum radius of $r_{ag}^{max} = 3.13$ (see Table J.3), there will be interference if standard full-depth teeth are used. Instead, as explained in Sample Problem 15.1D, we shall use a nonstandard addendum of $r_{ag} = r_g + 0.06 = 3.06$ and $r_{ap} = r_p + 0.29 = 1.29$.

(**c**) With this choice, one can now compute the contact ratio as illustrated in Table J.3.

Table J.3 *Solution to Sample Problem 15.1D*

Name	Symbol	Units	Relationship	Example
Number of teeth	N_p N_g			$N_p = 12$ $N_g = 36$
Nominal diametral pitch	P	per in.		$P = 6$
Nominal pressure angle	ϕ	degrees		$\phi = 20°$
Gear ratio	η		$\eta = N_g/N_p$	$\eta = 3$
Base width	b	in.	$9/P < b < 14/P$	$1.5 < b < 2.33$
Nominal pitch radius	r_p r_g	in.	$r_p = N_p/(2P)$ $r_g = N_g/(2P)$	$r_p = 1$ $r_g = 3$
Nominal base radius	r_{bp} r_{bg}	in.	$r_{bp} = r_p \cos\phi$ $r_{bg} = r_g \cos\phi$	$r_{bp} = 0.939$ $r_{bg} = 2.82$
Nominal center distance	c	in.	$c = \dfrac{(N_p + N_g)}{(2P)}$	$c = 4$
Maximum addendum radius to avoid interference	r_{ap}^{max} r_{ag}^{max}	in.	$r_{ap}^{max} = \sqrt{r_{bp}^2 + c^2 \sin^2\varphi}$ $r_{ag}^{max} = \sqrt{r_{bg}^2 + c^2 \sin^2\varphi}$	$r_{ap}^{max} = 1.66$ $r_{ag}^{max} = 3.13$
Standard addendum radius	r_{ap} r_{ag}	in.	$r_{ap} = (N_p + 2)/(2P)$ $r_{ag} = (N_g + 2)/(2P)$	$r_{ap} = 1.17$ $r_{ag} = 3.17$ (*Interference*)
Addendum radius	r_{ap} r_{ag}	in.	(*Non-standard*)	$r_{ap} = 1.29$ $r_{ag} = 3.06$
Standard dedendum radius	r_{dp} r_{dg}	in.	$r_{dp} = (N_p - 2.5)/(2P)$ $r_{dg} = (N_g - 2.5)/(2P)$ (*Standard*)	$r_{dp} = 0.792$ $r_{dg} = 2.792$
Nominal module	m	mm	$m = 25.4/P$	$m = 4.23$
Nominal circular pitch	p	in.	$p = \pi/P$	$p = 0.523$
Nominal tooth thickness	t	in.	$t = \pi/(2P)$	$t = 0.262$
Nominal contact ratio	CR		$\Delta_p = \sqrt{r_{ap}^2 - r_{bp}^2}$ $\Delta_g = \sqrt{r_{ag}^2 - r_{bg}^2}$ $CR = \dfrac{\Delta_p + \Delta_p - c \sin\phi}{p \cos\phi}$	$CR = 1.43$